$$\int \frac{P}{Q}dx = \frac{S}{U} + \int \frac{T}{V}dx \tag{1}$$

$$Q = (x - x_1)^{k_1} \dots (x - x_i)^{k_i} \left(x^2 + \alpha_1 x + \beta_1\right)^{m_1} \dots \left(x^2 + \alpha_j x + \beta_j\right)^{m_j} \tag{2}$$

$$U = (x - x_1)^{k_1 - 1} \dots (x - x_i)^{k_i - 1} \left(x^2 + \alpha_1 x + \beta_1\right)^{m_1 - 1} \dots \left(x^2 + \alpha_j x + \beta_j\right)^{m_j - 1} \tag{3}$$

$$V = (x - x_1) \dots (x - x_i) \left(x^2 + \alpha_1 x + \beta_1\right) \dots \left(x^2 + \alpha_j x + \beta_j\right) \tag{4}$$

$$UV = Q \tag{5}$$

$$\int \frac{dx}{\left(x^2+1\right)^3} = \frac{S}{\left(x^2+1\right)^2} + \int \frac{T}{\left(x^2+1\right)}dx \tag{6}$$

$$S = ax^3 + bx^2 + cx + d \tag{7}$$

$$T = px + q \tag{8}$$

$$\int \frac{dx}{\left(x^2+1\right)^3} = \frac{ax^3 + bx^2 + cx + d}{\left(x^2+1\right)^2} + \int \frac{px+q}{\left(x^2+1\right)}dx \tag{9}$$

$$\frac{1}{\left(x^2+1\right)^3} = \frac{3ax^2 + 2bx + c}{\left(x^2+1\right)^2} - 2\frac{ax^3 + bx^2 + cx + d}{\left(x^2+1\right)^3}2x + \frac{px+q}{\left(x^2+1\right)} \tag{10}$$

$$\frac{1}{\left(x^2+1\right)^3} = \frac{\left(3ax^2 + 2bx + c\right)\left(x^2+1\right)}{\left(x^2+1\right)^3} - 4x\frac{ax^3 + bx^2 + cx + d}{\left(x^2+1\right)^3} + \frac{(px+q)\left(x^2+1\right)^2}{\left(x^2+1\right)^3} \tag{11}$$

$$\left(3ax^2 + 2bx + c\right)\left(x^2+1\right) - 4x\left(ax^3 + bx^2 + cx + d\right) + (px+q)\left(x^2+1\right)^2 = 1 \tag{12}$$

$$\begin{cases} p = 0 \\ q - a = 0 \\ 2p - 2b = 0 \\ 2q - 3c + 3a = 0 \\ p - 4d + 2b = 0 \\ q + c = 1 \end{cases} \tag{13}$$

$$\begin{cases} a = \frac{3}{8} \\ b = 0 \\ c = \frac{5}{8} \\ d = 0 \\ p = 0 \\ q = \frac{3}{8} \end{cases} \tag{14}$$

$$\int \frac{dx}{\left(x^2+1\right)^3} = \frac{\frac{3}{8}x^3 + \frac{5}{8}x}{\left(x^2+1\right)^2} + \int \frac{\frac{3}{8}}{\left(x^2+1\right)} dx = \frac{1}{8}\frac{3x^3+5x}{\left(x^2+1\right)^2} + \frac{3}{8}\operatorname{arctg} x + C \tag{15}$$