1 10.25

$$\frac{1}{p^4+4} \risingdotseq ? \tag{1}$$

Разложим

$$\left(p^2+2\right)^2 = p^4+4p^2+4 \tag{2}$$

$$p^4+4 = \left(p^2+2\right)^2 - 4p^2 = \left(p^2+2p+2\right)\left(p^2-2p+2\right) = \left((p+1)^2+1\right)\left((p-1)^2+1\right) \tag{3}$$

О множителях

$$\frac{1}{p^2+1} \risingdotseq \sin t, \qquad F\left(p-p_0\right) \risingdotseq e^{p_0 t} f\left(t\right) \tag{4}$$

$$\frac{1}{(p-1)^2+1} \risingdotseq e^t \sin t \frac{1}{(p+1)^2+1} \risingdotseq e^{-t}\sin t \tag{5}$$

$$F\left(p\right)\cdot G\left(p\right) \risingdotseq \int\limits_0^t f\left(\tau\right) g\left(t-\tau\right) d\tau \tag{6}$$

$$\frac{1}{p^4+4} = \frac{1}{(p+1)^2+1}\cdot\frac{1}{(p-1)^2+1} \risingdotseq \int\limits_0^t e^{-\tau}\sin\tau e^{t-\tau}\sin\left(t-\tau\right)d\tau = \frac{e^t}{2}\int\limits_0^t e^{-2\tau}\left(\cos\left(2\tau-t\right)-\cos t\right)d\tau \tag{7}$$

1)

$$\int\limits_0^t e^{-2\tau}\cos\left(2\tau-t\right)d\tau = -\frac{1}{2}\int\limits_0^t \left(e^{-2\tau}\right)'\cos\left(2\tau-t\right)d\tau = -\frac{1}{2}e^{-2\tau}\cos\left(2\tau-t\right)\Big|_0^t - \int\limits_0^t e^{-2\tau}\sin\left(2\tau-t\right)d\tau = \tag{8}$$

$$= -\frac{1}{2}\left(e^{-2t}-1\right)\cos t + \frac{1}{2}\int\limits_0^t \left(e^{-2\tau}\right)'\sin\left(2\tau-t\right)d\tau = -\frac{1}{2}\left(e^{-2t}-1\right)\cos t + \frac{1}{2}e^{-2\tau}\sin\left(2\tau-t\right)\Big|_0^t - \int\limits_0^t e^{-2\tau}\cos\left(2\tau-t\right)d\tau$$

$$\int\limits_0^t e^{-2\tau}\cos\left(2\tau-t\right)d\tau = -\frac{1}{4}\left(e^{-2t}-1\right)\cos t + \frac{1}{4}\left(e^{-2t}+1\right)\sin t \tag{9}$$

2)

$$\int\limits_0^t e^{-2\tau}\cos t d\tau = \cos t\ \frac{e^{-2\tau}}{-2}\Big|_0^t = -\cos t\frac{e^{-2t}-1}{2} \tag{10}$$

Итого

$$\begin{aligned} \frac{1}{p^4+4} &\risingdotseq \frac{e^t}{2}\int\limits_0^t e^{-2\tau}\left(\cos\left(2\tau-t\right)-\cos t\right)d\tau = \frac{e^t}{2}\left[\int\limits_0^t e^{-2\tau}\cos\left(2\tau-t\right)d\tau - \int\limits_0^t e^{-2\tau}\cos t d\tau\right] = \\ &= \frac{e^t}{2}\left[-\frac{1}{4}\left(e^{-2t}-1\right)\cos t + \frac{1}{4}\left(e^{-2t}+1\right)\sin t + \cos t\frac{e^{-2t}-1}{2}\right] = \frac{e^t}{2}\left[\frac{e^{-2t}+1}{4}\sin t + \cos t\frac{e^{-2t}-1}{4}\right] = \\ &= \frac{1}{4}\left[\frac{e^{-t}+e^t}{2}\sin t + \cos t\frac{e^{-t}-e^t}{2}\right] = \frac{1}{4}\left(\operatorname{ch} t\sin t - \operatorname{sh} t\cos t\right) \end{aligned} \tag{11}$$

2 11.16

$$x'' - 9x = \operatorname{sh} t, \quad x(0) = -1,\ x'(0) = 3 \tag{12}$$

$$x \risingdotseq X, \quad x' \risingdotseq pX - x(0), \quad x'' \risingdotseq p(pX - x(0)) - x'(0) = p^2X - px(0) - x'(0) \tag{13}$$

$$x'' \risingdotseq p^2X + p - 3 \tag{14}$$

$$\operatorname{sh} t = \frac{e^t - e^{-t}}{2} \quad e^{qt} \risingdotseq \frac{1}{p-q} \quad \operatorname{sh} qt \risingdotseq \frac{1}{2}\left(\frac{1}{p-q} - \frac{1}{p+q}\right) = \frac{q}{p^2 - q^2} \tag{15}$$

$$p^2X + p - 3 - 9X = \frac{1}{p^2 - 1} \tag{16}$$

$$\left(p^2 - 9\right)X = \frac{1}{p^2 - 1} - p + 3 \tag{17}$$

$$X = \frac{1}{(p^2-1)(p^2-9)} - \frac{p-3}{p^2-9} = \frac{1}{(p^2-1)(p^2-9)} - \frac{1}{p+3} \tag{18}$$

$$\frac{1}{p+3} \risingdotseq e^{-3t} \tag{19}$$

$$\frac{1}{(p^2-1)(p^2-9)} = \frac{A}{p^2-1} + \frac{B}{p^2-9} = \frac{A\left(p^2-9\right) + B\left(p^2-1\right)}{(p^2-1)(p^2-9)} \tag{20}$$

$$\begin{cases} A + B = 0 \\ -9A - B = 1 \end{cases} \tag{21}$$

$$A = -\frac{1}{8}, \quad B = \frac{1}{8} \tag{22}$$

$$\frac{1}{(p^2-1)(p^2-9)} = -\frac{1}{8}\frac{1}{p^2-1} + \frac{1}{24}\frac{3}{p^2-9} \risingdotseq -\frac{1}{8}\operatorname{sh} t + \frac{1}{24}\operatorname{sh} 3t \tag{23}$$

$$x = -\frac{1}{8}\operatorname{sh} t + \frac{1}{24}\operatorname{sh} 3t - e^{-3t} \tag{24}$$

$$\operatorname{ch} t - \operatorname{sh} t = \frac{e^t + e^{-t}}{2} - \frac{e^t - e^{-t}}{2} = e^{-t} \qquad e^{-3t} = \operatorname{ch} 3t - \operatorname{sh} 3t \tag{25}$$

$$x = -\frac{1}{8}\operatorname{sh} t + \frac{1}{24}\operatorname{sh} 3t - (\operatorname{ch} 3t - \operatorname{sh} 3t) = \frac{25}{24}\operatorname{sh} 3t - \operatorname{ch} 3t - \frac{1}{8}\operatorname{sh} t \tag{26}$$

3 11.21

$$x' + x = f(t), \qquad f(t) = \begin{cases} 1, & 0 \leqslant t < 2 \\ 0, & t \geqslant 2 \end{cases} \tag{27}$$

$$x(0) = 0 \tag{28}$$

$$f(t) \risingdotseq F(p) = \int\limits_0^\infty f(t)\, e^{-pt} dt = \int\limits_0^2 f(t)\, e^{-pt} dt + \int\limits_2^\infty f(t)\, e^{-pt} dt = \int\limits_0^2 e^{-pt} dt + 0 = \tag{29}$$

$$= -\frac{1}{p}e^{-pt}\Big|_0^2 = -\frac{1}{p}\left(e^{-2p}-1\right) = \frac{1-e^{-2p}}{p} \tag{30}$$

$$x' \risingdotseq pX - x(0) = pX \tag{31}$$

$$pX + X = \frac{1-e^{-2p}}{p} \tag{32}$$

$$X = \frac{1-e^{-2p}}{p(p+1)} \tag{33}$$

$$e^{-t} \risingdotseq \frac{1}{p+1} \tag{34}$$

$$\int\limits_0^t e^{-\tau}d\tau \cdot \eta(t) = -\,e^{-\tau}\big|_0^t \cdot \eta(t) = \left(1-e^{-t}\right)\cdot\eta(t) \risingdotseq \frac{1}{p(p+1)} \tag{35}$$

$$\left(1-e^{-(t-\tau)}\right)\cdot\eta(t-\tau) \risingdotseq e^{-p\tau}\frac{1}{p(p+1)} \tag{36}$$

$$\left(1-e^{-(t-2)}\right)\cdot\eta(t-2) \risingdotseq \frac{e^{-2p}}{p(p+1)} \tag{37}$$

$$\left(1-e^{-t}\right)\cdot\eta(t) - \left(1-e^{-(t-2)}\right)\cdot\eta(t-2) \risingdotseq \frac{1}{p(p+1)} - \frac{e^{-2p}}{p(p+1)} = \frac{1-e^{-2p}}{p(p+1)} \tag{38}$$

$$x(t) = \left(1-e^{-t}\right)\cdot\eta(t) - \left(1-e^{-(t-2)}\right)\cdot\eta(t-2) \tag{39}$$

$$t > 2 \quad \Rightarrow \quad x(t) = e^{2-t} - e^{-t} \tag{40}$$

4 11.45

$$4\int\limits_0^t (\tau - t)\,x(\tau)\,d\tau = x(t) - t \tag{41}$$

$$1 \risingdotseq \frac{1}{p}, \quad -tf(t) \risingdotseq F'(p), \quad -t \risingdotseq -\frac{1}{p^2}, \quad t \risingdotseq \frac{1}{p^2} \tag{42}$$

$$\int\limits_0^t f(\tau)\,g(t-\tau)\,d\tau \risingdotseq F(p)\,G(p) \tag{43}$$

$$4\int\limits_0^t (\tau - t)\,x(\tau)\,d\tau = -4\int\limits_0^t (t-\tau)\,x(\tau)\,d\tau \tag{44}$$

$$f(t) = x(t), \quad g(t) = t \tag{45}$$

$$F(p) = X(p), \quad G(p) = \frac{1}{p^2} \tag{46}$$

Изображение уравнения:

$$-4\frac{1}{p^2}X\left(p\right) = X\left(p\right) - \frac{1}{p^2} \tag{47}$$

$$-4X\left(p\right) = p^2X\left(p\right) - 1 \tag{48}$$

Изображение решения:

$$X\left(p\right) = \frac{1}{p^2+4} \tag{49}$$

Само решение:

$$x\left(t\right) = \frac{1}{2}\sin 2t \tag{50}$$