1)

$$\sqrt{ax^2+bx+c} = \pm\sqrt{a}x + z \qquad (a>0) \tag{1}$$

$$\cancel{ax^2} + bx + c = \cancel{ax^2} \pm 2\sqrt{a}xz + z^2 \tag{2}$$

$$bx \mp 2\sqrt{a}xz = z^2 - c \tag{3}$$

$$x = \frac{z^2-c}{b \mp 2\sqrt{a}z} \tag{4}$$

$$dx = \frac{(2z)\left(b \mp 2\sqrt{a}z\right) - \left(z^2-c\right)\left(\mp 2\sqrt{a}\right)}{\left(b \mp 2\sqrt{a}z\right)^2}dz = \frac{\left(2bz \mp 4\sqrt{a}z^2\right) \pm 2\sqrt{a}\left(z^2-c\right)}{\left(b \mp 2\sqrt{a}z\right)^2}dz =$$

$$= \mp 2\frac{\sqrt{a}z^2 \pm bz + \sqrt{a}c}{\left(b \mp 2\sqrt{a}z\right)^2}dz \tag{5}$$

2)

$$\sqrt{ax^2+bx+c} = xz \pm \sqrt{c} \qquad (c>0) \tag{6}$$

$$ax^2 + bx + \cancel{c} = x^2z^2 \pm 2\sqrt{c}xz + \cancel{c} \tag{7}$$

$$ax + b = xz^2 \pm 2\sqrt{c}z \tag{8}$$

$$ax - xz^2 = \pm 2\sqrt{c}z - b \tag{9}$$

$$x = \frac{\pm 2\sqrt{c}z - b}{a - z^2} \tag{10}$$

$$dx = \frac{\left(\pm 2\sqrt{c}\right)\left(a-z^2\right) - \left(\pm 2\sqrt{c}z - b\right)(-2z)}{\left(a-z^2\right)^2}dz = \frac{\pm 2\sqrt{c}\left(a-z^2\right) + 2z\left(\pm 2\sqrt{c}z - b\right)}{\left(a-z^2\right)^2}dz =$$

$$= \frac{\pm 2\sqrt{c}a \pm 2\sqrt{c}z^2 - 2zb}{\left(a-z^2\right)^2}dz = \pm 2\frac{\sqrt{c}a + \sqrt{c}z^2 \mp zb}{\left(a-z^2\right)^2}dz \tag{11}$$

3)

$$\sqrt{a\left(x-x_1\right)\left(x-x_2\right)} = z\left(x-x_1\right) \qquad \left(b^2 - 4ac > 0\right) \tag{12}$$

$$a\cancel{\left(x-x_1\right)}\left(x-x_2\right) = z^2\left(x-x_1\right)^{\cancel{2}} \tag{13}$$

$$ax - z^2x = ax_2 - x_1z^2 \tag{14}$$

$$x = \frac{ax_2 - x_1z^2}{a - z^2} \tag{15}$$

$$dx = \frac{\left(-x_1 2z\right)\left(a-z^2\right) - \left(ax_2 - x_1z^2\right)(-2z)}{\left(a-z^2\right)^2}dz = 2z\frac{\left(ax_2 - x_1z^2\right) - x_1\left(a-z^2\right)}{\left(a-z^2\right)^2}dz =$$

$$= 2z\frac{ax_2 \cancel{- x_1z^2} - ax_1 \cancel{+ x_1z^2}}{\left(a-z^2\right)^2}dz = 2az\frac{x_2 - x_1}{\left(a-z^2\right)^2}dz \tag{16}$$

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$$\int \frac{x - \sqrt{x^2+3x+2}}{x + \sqrt{x^2+3x+2}}dx \tag{17}$$

1) (тупиковый)

$$\sqrt{x^2+3x+2} = -x + z \tag{18}$$

$$x^2 + 3x + 2 = x^2 - 2xz + z^2 \tag{19}$$

$$3x + 2 = -2xz + z^2 \tag{20}$$

$$3x + 2xz = z^2 - 2 \tag{21}$$

$$x = \frac{z^2-2}{3+2z} \tag{22}$$

$$dx = \frac{2z(3+2z) - 2\left(z^2-2\right)}{(3+2z)^2}dz = \frac{6z + 4z^2 - 2z^2 + 4}{(3+2z)^2}dz = \frac{2z^2+6z+4}{(3+2z)^2}dz \tag{23}$$

$$x + \sqrt{x^2+3x+2} = x - x + z = z \tag{24}$$

$$x - \sqrt{x^2+3x+2} = x + x - z = 2\frac{z^2-2}{3+2z} - z \tag{25}$$

$$\int \frac{x - \sqrt{x^2+3x+2}}{x + \sqrt{x^2+3x+2}}dx = \int \frac{2\frac{z^2-2}{3+2z} - z}{z}\frac{2z^2+6z+4}{(3+2z)^2}dz = \int \frac{2\left(z^2-2\right) - z(3+2z)}{z(3+2z)}\frac{2z^2+6z+4}{(3+2z)^2}dz =$$

$$= \int \frac{\cancel{2z^2} - 4 - 3z - \cancel{2z^2}}{z\left(3+2z\right)} \frac{2z^2+6z+4}{\left(3+2z\right)^2} dz = -2 \int \frac{3z+4}{z\left(3+2z\right)} \frac{z^2+3z+2}{\left(3+2z\right)^2} dz \tag{26}$$

Вариант 3)

$$\left(x+2\right)\left(x+1\right) = x^2+3x+2 \tag{27}$$

$$\sqrt{x^2+3x+2} = z\left(x+1\right) \tag{28}$$

$$z = \frac{\sqrt{x^2+3x+2}}{x+1} \tag{29}$$

$$\left(x+2\right)\left(x+1\right) = z\left(x+1\right)^2 \tag{30}$$

$$x+2 = zx+z \tag{31}$$

$$x = \frac{z-2}{1-z} = \frac{z-1-1}{1-z} = -1 + \frac{1}{z-1} \tag{32}$$

$$dx = -\frac{dz}{\left(z-1\right)^2} \tag{33}$$

$$\int \frac{x-\sqrt{x^2+3x+2}}{x+\sqrt{x^2+3x+2}} dx = -\int \frac{x-z\left(x+1\right)}{x+z\left(x+1\right)} \frac{dz}{\left(z-1\right)^2} = -\int \frac{\left(1-z\right)x-z}{\left(1+z\right)x+z} \frac{dz}{\left(z-1\right)^2} = \tag{34}$$

$$= -\int \frac{\left(1-z\right)\frac{z-2}{1-z}-z}{\left(1+z\right)\frac{z-2}{1-z}+z} \frac{dz}{\left(z-1\right)^2} = -\int \frac{z-2-z}{-\left(1+z\right)\left(z-2\right)+z\left(z-1\right)} \frac{dz}{z-1} = \int \frac{2}{-\left(z^2-z-2\right)+z^2-z} \frac{dz}{z-1} =$$

$$= \int \frac{2}{\cancel{-z^2}\cancel{+z}+2\cancel{+z^2}\cancel{-z}} \frac{dz}{z-1} = \int \frac{dz}{z-1} = \ln\left|z-1\right| + C = \ln\left|\frac{\sqrt{x^2+3x+2}}{x+1} - 1\right| + C$$