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Условия:

$$x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} = 0 \tag{1}$$

$$\left\{\begin{array}{l} u = xy \\ v = \frac{x}{y} \end{array}\right. \tag{2}$$

x)

$$\frac{\partial z}{\partial x} = z'_u u'_x + z'_v v'_x = z'_u y + z'_v \frac{1}{y} \tag{3}$$

$$\begin{aligned}\frac{\partial^2 z}{\partial x^2} &= \frac{\partial}{\partial x}\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(z'_u y + z'_v\frac{1}{y}\right) = y\frac{\partial}{\partial x}z'_u + \frac{1}{y}\frac{\partial}{\partial x}z'_v = \left(z''_{uu}u'_x + z''_{uv}v'_x\right)y + \left(z''_{vu}u'_x + z''_{vv}v'_x\right)\frac{1}{y} = \\ &= \left(z''_{uu}y + z''_{uv}\frac{1}{y}\right)y + \left(z''_{vu}y + z''_{vv}\frac{1}{y}\right)\frac{1}{y} = z''_{uu}y^2 + 2z''_{uv} + z''_{vv}\frac{1}{y^2}\end{aligned} \tag{4}$$

y)

$$\frac{\partial z}{\partial y} = z'_u u'_y + z'_v v'_y = z'_u x - z'_v\frac{x}{y^2} \tag{5}$$

$$\begin{aligned}\frac{\partial^2 z}{\partial y^2} &= \frac{\partial}{\partial y}\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(z'_u x - z'_v\frac{x}{y^2}\right) = x\frac{\partial}{\partial y}z'_u - \frac{x}{y^2}\frac{\partial}{\partial y}z'_v + 2z'_v\frac{x}{y^3} = \\ &= x\left(z''_{uu}x - z''_{uv}\frac{x}{y^2}\right) - \frac{x}{y^2}\left(z''_{vu}x - z''_{vv}\frac{x}{y^2}\right) + 2z'_v\frac{x}{y^3} = \\ &= z''_{uu}x^2 - 2z''_{uv}\frac{x^2}{y^2} + z''_{vv}\frac{x^2}{y^4} + 2z'_v\frac{x}{y^3}\end{aligned} \tag{6}$$

Подставляем:

$$x^2\left(z''_{uu}y^2 + 2z''_{uv} + z''_{vv}\frac{1}{y^2}\right) - y^2\left(z''_{uu}x^2 - 2z''_{uv}\frac{x^2}{y^2} + z''_{vv}\frac{x^2}{y^4} + 2z'_v\frac{x}{y^3}\right) = 0 \tag{7}$$

$$\cancel{z''_{uu}x^2y^2} + 2z''_{uv}x^2 + \cancel{z''_{vv}\frac{x^2}{y^2}} - \cancel{z''_{uu}x^2y^2} + 2z''_{uv}x^2 - \cancel{z''_{vv}\frac{x^2}{y^2}} - 2z'_v\frac{x}{y} = 0 \tag{8}$$

$$4z''_{uv}x^2 - 2z'_v\frac{x}{y} = 0 \tag{9}$$

$$2z''_{uv}uv - z'_v v = 0 \tag{10}$$

$$2u\,z''_{uv} - z'_v = 0 \tag{11}$$

Решаем:

$$\frac{\frac{\partial}{\partial u}z'_v}{z'_v} = \frac{1}{2u}, \quad \Rightarrow \quad \frac{\partial}{\partial u}\ln|z'_v| = \frac{1}{2}\frac{\partial}{\partial u}\ln|u|, \quad \Rightarrow \quad \frac{\partial}{\partial u}\ln\frac{|z'_v|}{\sqrt{|u|}} = 0, \tag{12}$$

$$\ln\frac{|z'_v|}{\sqrt{|u|}} = \tilde{f}(v), \quad \Rightarrow \quad |z'_v| = e^{\tilde{f}(v)}\sqrt{|u|}, \quad \Rightarrow \quad z'_v = \pm e^{\tilde{f}(v)}\sqrt{|u|} \equiv f(v)\sqrt{|u|} \tag{13}$$

$$z = F(v)\sqrt{|u|} + G(u) = F\left(\frac{x}{y}\right)\sqrt{|xy|} + G(xy). \tag{14}$$

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$$y\frac{\partial^2 z}{\partial y^2}+2\frac{\partial z}{\partial y}=\frac{2}{x} \tag{15}$$

$$\begin{cases} u=\dfrac{x}{y} \\ v=x \\ w=xz-y \end{cases} \tag{16}$$

1)

$$\begin{cases} u'_y=-\dfrac{x}{y^2} \\ v'_y=0 \\ w'_y=xz'_y-1=w'_u u'_y+w'_v v'_y \end{cases} \tag{17}$$

$$xz'_y-1=-\frac{x}{y^2}w'_u \tag{18}$$

$$z'_y=\frac{1}{x}-\frac{1}{y^2}w'_u \tag{19}$$

2)

$$z''_{yy}=\frac{\partial}{\partial y}z'_y=\frac{\partial}{\partial y}\left(\frac{1}{x}-\frac{1}{y^2}w'_u\right)=\frac{2}{y^3}w'_u-\frac{1}{y^2}\frac{\partial}{\partial y}w'_u= \tag{20}$$
$$=\frac{2}{y^3}w'_u-\frac{1}{y^2}\left(w''_{uu}u'_y+w''_{uv}v'_y\right)=\frac{2}{y^3}w'_u+\frac{x}{y^4}w''_{uu}$$

Подставляем в уравнение (15):

$$y\left(\frac{2}{y^3}w'_u+\frac{x}{y^4}w''_{uu}\right)+2\left(\frac{1}{x}-\frac{1}{y^2}w'_u\right)=\frac{2}{x} \tag{21}$$

$$\cancel{\frac{2}{y^2}w'_u}+\frac{x}{y^3}w''_{uu}+\cancel{\frac{2}{x}}-\cancel{\frac{2}{y^2}w'_u}=\cancel{\frac{2}{x}} \tag{22}$$

$$\frac{x}{y^3}w''_{uu}=0 \tag{23}$$

$$w''_{uu}=0 \tag{24}$$

$$w''_u=f(v) \tag{25}$$

$$w=\varphi(v)\,u+\psi(v) \tag{26}$$

$$z=\frac{1}{x}(w+y)=\frac{1}{x}\left[\varphi(v)\,u+\psi(v)+y\right]=\frac{1}{x}\left[\varphi(x)\,\frac{x}{y}+\psi(x)+y\right] \tag{27}$$