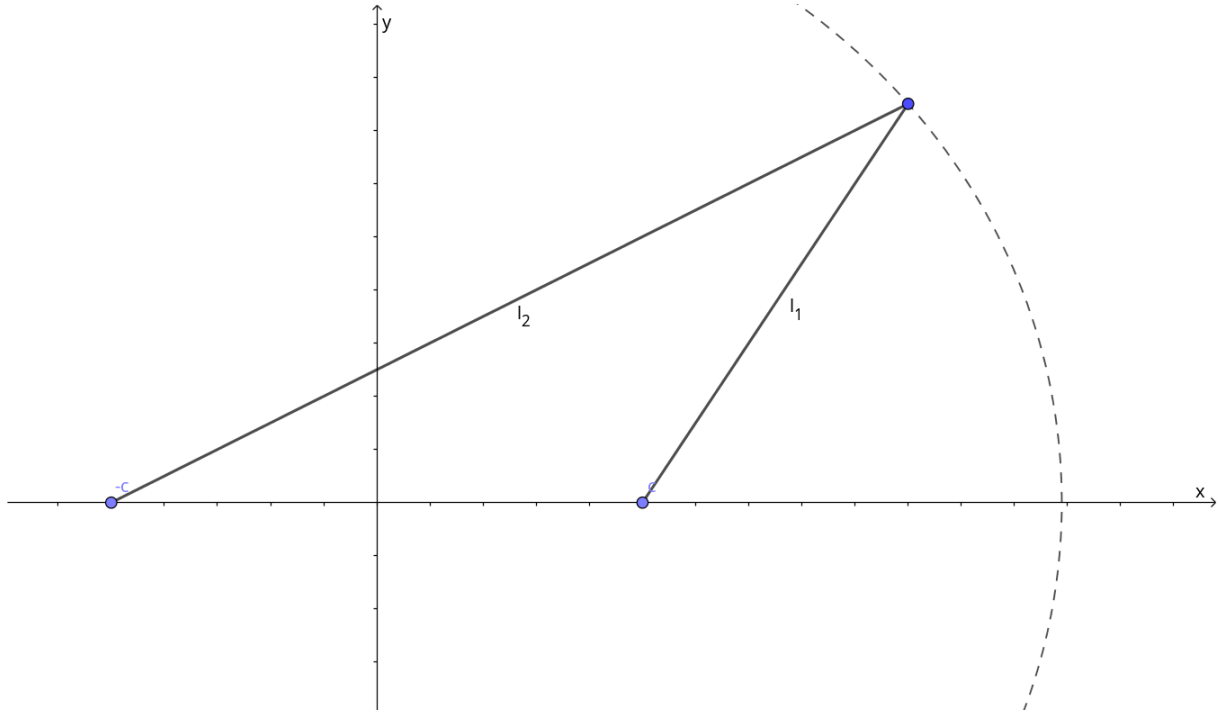


# 1 Эллипс

## 1.1 Каноническая формула



Сумма расстояний до двух точек

$$l_1 = \sqrt{(x - c)^2 + y^2}, \quad l_2 = \sqrt{(x + c)^2 + y^2} \quad (1)$$

$$l_1 + l_2 = 2L \quad (2)$$

$$\sqrt{(x - c)^2 + y^2} + \sqrt{(x + c)^2 + y^2} = 2L \quad (3)$$

$$\sqrt{(x + c)^2 + y^2} = 2L - \sqrt{(x - c)^2 + y^2} \quad (4)$$

$$(x + c)^2 + y^2 = 4L^2 - 4L\sqrt{(x - c)^2 + y^2} + (x - c)^2 + y^2 \quad (5)$$

$$(x + c)^2 - (x - c)^2 - 4L^2 = 4xc - 4L^2 = -4L\sqrt{(x - c)^2 + y^2} \quad (6)$$

$$L^2 - xc = L\sqrt{(x - c)^2 + y^2} \quad (7)$$

$$L^4 - 2L^2xc + x^2c^2 = L^2[x^2 - 2cx + c^2 + y^2] \quad (8)$$

$$L^4 - \overbrace{2L^2xc}^{\leftarrow} + \overbrace{x^2c^2}^{\leftarrow} = L^2x^2 - \overbrace{2L^2cx}^{\leftarrow} + \overbrace{L^2c^2}^{\leftarrow} + L^2y^2 \quad (9)$$

$$L^4 - L^2c^2 = L^2x^2 - x^2c^2 + L^2y^2 \quad (10)$$

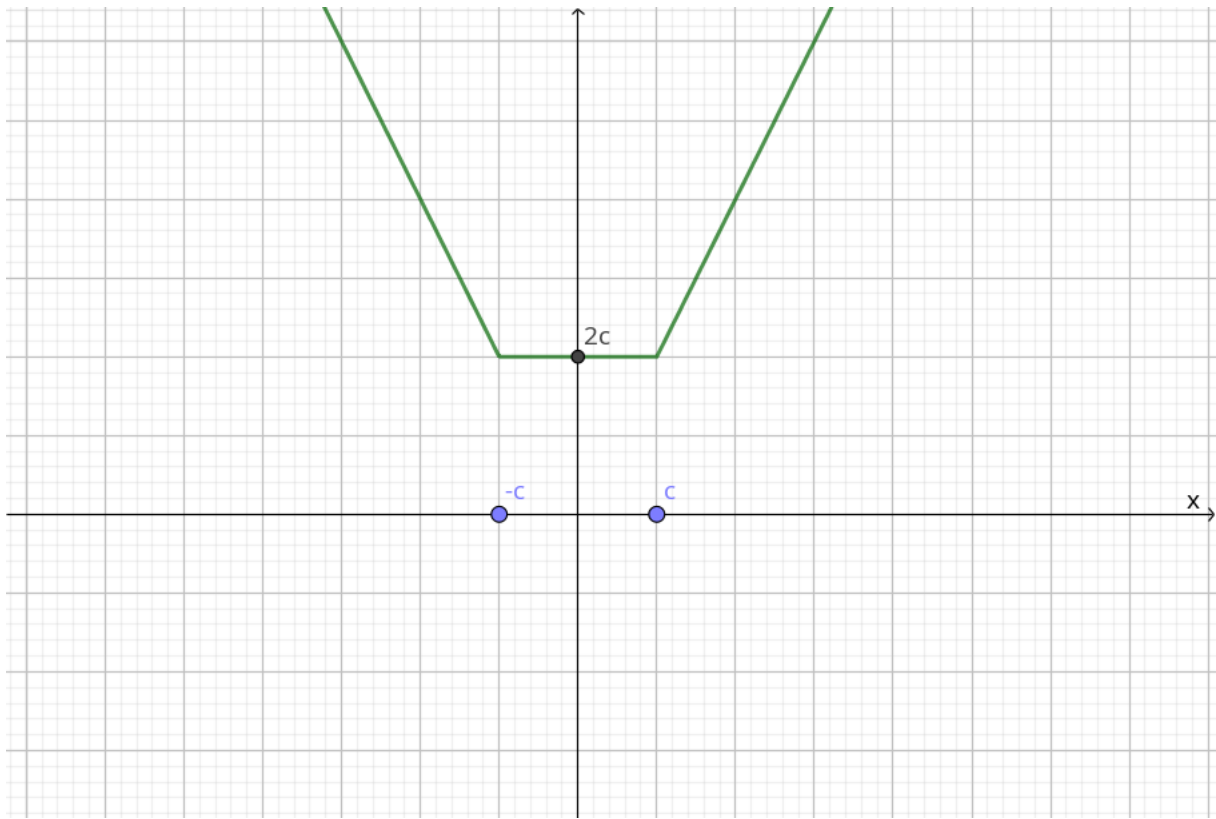
$$(L^2 - c^2)x^2 + L^2y^2 = (L^2 - c^2)L^2 \left| \cdot \frac{1}{L^2(L^2 - c^2)} \right. \quad (11)$$

$$\frac{x^2}{L^2} + \frac{y^2}{L^2 - c^2} = 1 \quad (12)$$

## 1.2 Положительность $L^2 - c^2$

$$l_1 = \sqrt{(x - c)^2 + y^2} \geq |x - c|, \quad l_2 = \sqrt{(x + c)^2 + y^2} \geq |x + c| \quad (13)$$

$$2L = l_1 + l_2 \geq |x + c| + |x - c| \geq 2c \quad (14)$$



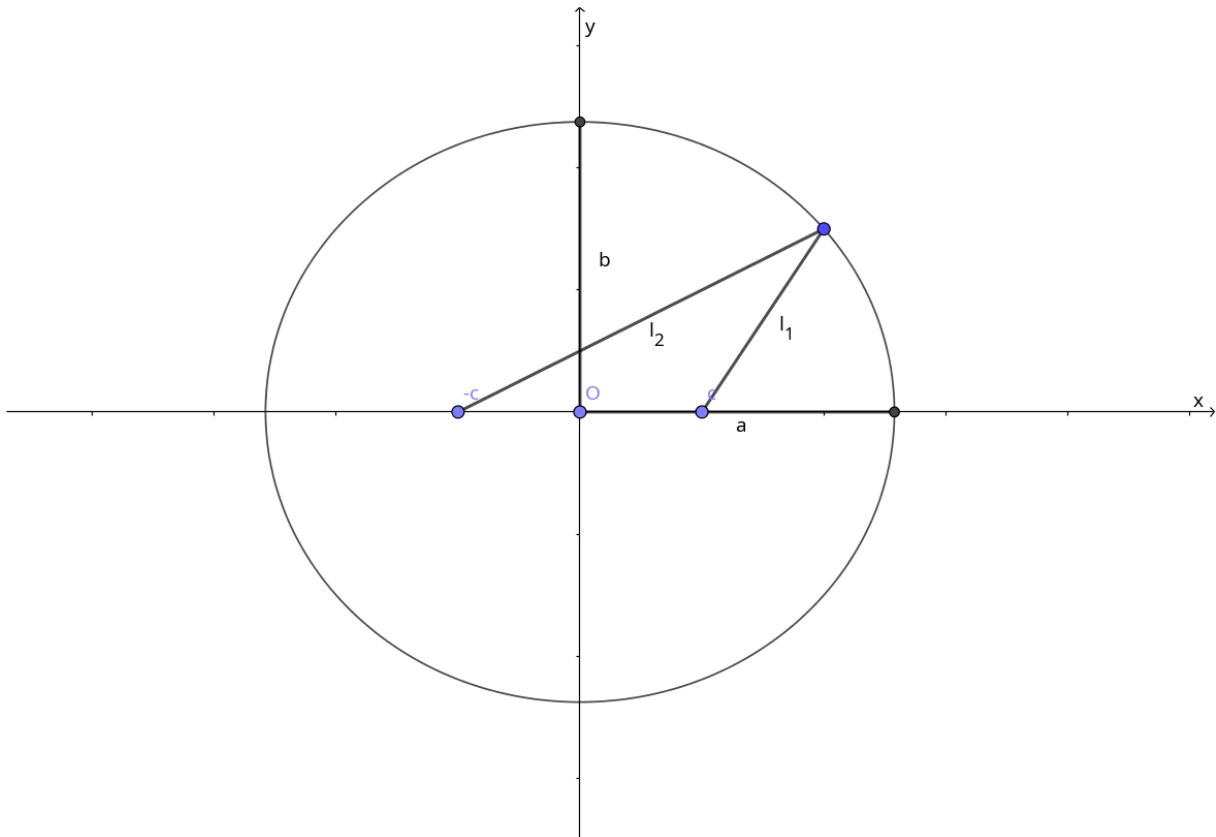
$$L^2 - c^2 \geq 0 \tag{15}$$

При  $L = c$   $y = 0$ .

Переобозначим  $L \equiv a$ ,  $\sqrt{L^2 - c^2} \equiv b$ ,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{16}$$

$$-a \leq x \leq a, -b \leq y \leq b.$$



### 1.3 Эксцентриситет

$x > 0$

$$y^2 = b^2 \left(1 - \frac{x^2}{a^2}\right) \quad (17)$$

$$l_1 = \sqrt{(x-c)^2 + y^2} = \sqrt{(x-c)^2 + b^2 \left(1 - \frac{x^2}{a^2}\right)} = \sqrt{x^2 - 2xc + c^2 + b^2 - \frac{b^2}{a^2}x^2} =$$

$L^2 - c^2 = a^2 - c^2 = b^2$ . Отсюда  $b^2 + c^2 = a^2$ ,  $a^2 - b^2 = c^2$

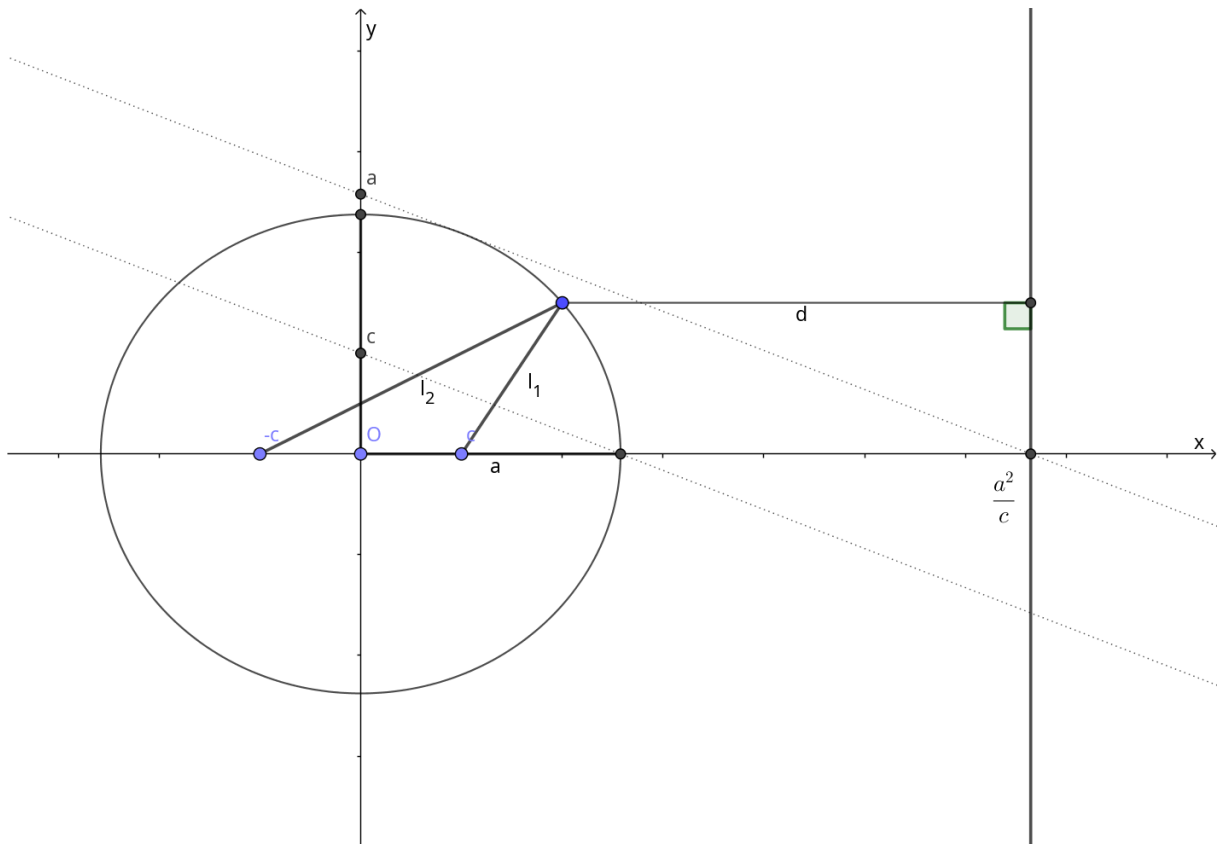
$$= \sqrt{\frac{a^2 - b^2}{a^2}x^2 - 2xc + c^2 + b^2} = \sqrt{\frac{c^2}{a^2}x^2 - 2xc + a^2} = \sqrt{\left(\frac{c}{a}x - a\right)^2} = \left|\frac{c}{a}x - a\right| = \quad (18)$$

$c^2 = a^2 - b^2 < a^2$ ,  $c < a$ ,  $1 < \frac{a}{c}$ ,  $x \leq a < \frac{a^2}{c}$ ,  $x - \frac{a^2}{c} < 0$

$$= \frac{c}{a} \left|x - \frac{a^2}{c}\right| = \frac{c}{a} \left(\frac{a^2}{c} - x\right)$$

$\frac{a^2}{c} - x \equiv d$

$$\frac{l_1}{d} = \frac{c}{a} \equiv \varepsilon < 1 \quad (19)$$



## 2 Парабола

Пусть

$$\frac{l_1}{d} = \varepsilon = 1, \quad l_1 = d \quad (20)$$

Директриса - ось  $y$ , тогда  $d = x$ :

$$\sqrt{(x-c)^2 + y^2} = x \quad (21)$$

$$(x-c)^2 + y^2 = x^2 \quad (22)$$

$$x^2 - 2xc + c^2 + y^2 = x^2 \quad (23)$$

$$c^2 + y^2 = 2cx \quad (24)$$

$$x = \frac{y^2 + c^2}{2c} = \frac{y^2}{2c} + \frac{c}{2} \quad (25)$$

